2002 – Linear Motion Question

1. **(a)** A stone is thrown vertically upwards under gravity with a speed of u m/s from a point 30 metres above the horizontal ground. The stone hits the ground 5 seconds later.

(i) Find the value of u.

(ii) Find the speed with which the stone hits the ground.

(b) A particle, with initial speed u, moves in a straight line with constant acceleration.
During the time interval from 0 to t, the particle travels a distance p.
During the time interval from t to $2t$, the particle travels a distance q.
During the time interval from $2t$ to $3t$, the particle travels a distance r.

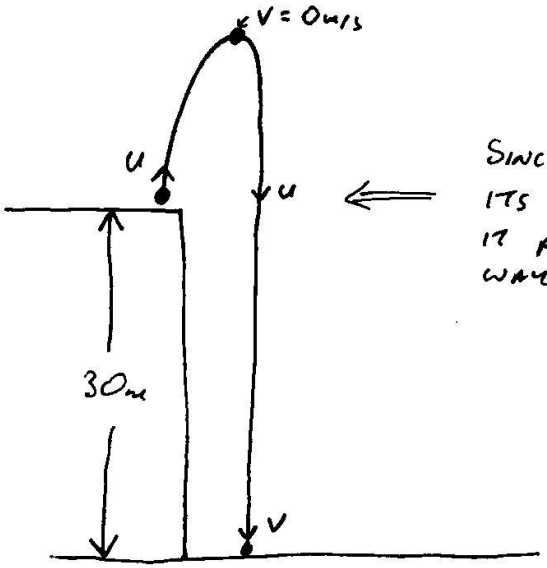
(i) Show that $2q = p + r$.

(ii) Show that the particle travels a further distance $2r - q$ in the time interval from $3t$ to $4t$.

QUESTION 1

2002

Q1/(a)



SINCE THE MOTION IS SYMMETRICAL ITS SPEED IS u AGAIN WHEN IT PASSES THE LEDGE ON THE WAY DOWN.

- FIND THE TIME TO THE TOP:

$u = u$
$v = 0$
$a = -9.8$
$s = -$
$t = ?$

$v = u + at$

$0 = u - 9.8t$

$\boxed{\frac{u}{9.8} = t}$ = TIME FROM LEDGE TO THE TOP

- TIME FROM LEDGE TO THE GROUND ON THE WAY DOWN:

$u = u$
$v = v$
$a = 9.8$
$s = 30m$
$t = ?$ ←

BUT TOTAL TIME IS 5 SEC.
SO, TIME TO GROUND = $5 - 2\left(\frac{u}{9}\right)$

$t = 5 - \frac{2u}{9}$

$$s = ut + \frac{1}{2}at^2$$

$$30 = u\left(5 - \frac{2u}{9}\right) + \frac{1}{2}(9)\left(5 - \frac{2u}{9}\right)^2$$

$$30 = 5u - \frac{2u^2}{g} + \frac{g}{2}\left[25 - \frac{10u}{g} - \frac{10u}{g} + \frac{4u^2}{g^2}\right]$$

$$30 = 5u - \frac{2u^2}{g} + \frac{25g}{2} - 5u - 5u + \frac{2u^2}{g}$$

$$30 = \frac{25g}{2} - 5u$$

$$5u = \frac{25g}{2} - 30$$

$$5u = \frac{25(9.8)}{2} - 30$$

$$5u = 92.5 \Rightarrow \underline{\underline{u = 18.5 \text{ m/s}}}$$

(ii) FIND SPEED ON HITTING GROUND:

- TRAVELLING AT SPEED u WHEN IT PASSES THE LEDGE ON THE WAY DOWN.

$u = 18.5$
$v = ?$
$a = 9.8$
$s = 30$ m
$t = -$

$$v^2 = u^2 + 2as$$

$$v^2 = (18.5)^2 + 2(9.8)(30)$$

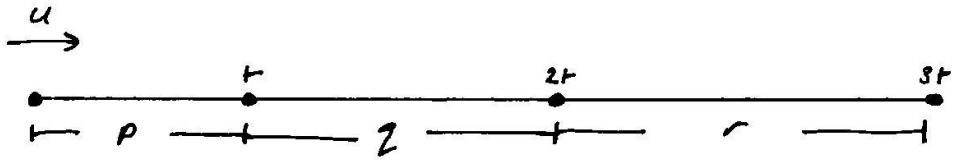
$$v^2 = 342.25 + 588$$

$$v^2 = 930.25$$

$$\underline{\underline{v = 30.5 \text{ m/s}}}$$

2002

Q/ (b) (i)



0 → t sec

$u = u$
$v = -$
$a = a$
$s = p$
$t = t$

$s = ut + \frac{1}{2}at^2$

$p = ut + \frac{1}{2}at^2$

$\boxed{2p = 2ut + at^2}$ I

0 → 2t sec

$u = u$
$v = -$
$a = a$
$s = p+q$
$t = 2t$

$s = ut + \frac{1}{2}at^2$

$p+q = u(2t) + \frac{1}{2}(a)(2t)^2$

$\boxed{p+q = 2ut + 2at^2}$ II

0 → 3t sec

$u = u$
$v = -$
$a = a$
$s = p+q+r$
$t = 3t$

$s = ut + \frac{1}{2}at^2$

$p+q+r = u(3t) + \frac{1}{2}(a)(3t)^2$

$\boxed{p+q+r = 3ut + \frac{9}{2}at^2}$ III

Q1 (b) From eqn I: $at^2 = 2p - 2ut$

Sub into II:

$$p + q = 2ut + 2at^2$$

$$p + q = 2ut + 2(2p - 2ut)$$

$$p + q = 2ut + 4p - 4ut$$

$$p + q = 4p - 2ut$$

$$-3p + q = -2ut$$

so, $\boxed{3p - q = 2ut}$

Sub into III: $p + q + r = 3ut + \frac{9}{2}at^2$

$$\Rightarrow p + q + r = 3ut + \frac{9}{2}(2p - 2ut)$$

$$p + q + r = 3ut + 9p - 9ut$$

$$p + q + r = 9p - 6ut$$

But, $2ut = 3p - q$

so, $p + q + r = 9p - 3(2ut)$

$$p + q + r = 9p - 3(3p - q)$$

$$p + q + r = 9p - 9p + 3q$$

$$p + q + r = 3q$$

so, $\underline{\underline{p + r = 2q}}$

Q

Q1

(5)

(ii) PARTICLES TRAVELS DIST. X DURING $3t \to 4t$

$\boxed{0 \to 4t}$

$u = u$
$v = -$
$a = a$
$s = p+q+r+x$
$t = 4t$

$s = ut + \frac{1}{2}at^2$

$p+q+r+x = u(4t) + \frac{1}{2}a(4t)^2$

$p+q+r+x = 4ut + 8at^2$

(But $at^2 = 2p - 2ut$)

so

$p+q+r+x = 4ut + 8(2p - 2ut)$

$p+q+r+x = 4ut + 16p - 16ut$

$-15p + q + r + x = -12ut$

$-15p + q + r + x = -6(2ut)$

(But $2ut = 3p - q$)

$-15p + q + r + x = -6(3p - q)$

$-15p + q + r + x = -18p + 6q$

$3p - 5q + r + x = 0$

(But $p = 2q - r$)

$3(2q - r) - 5q + r + x = 0$

$6q - 3r - 5q + r + x = 0$

$q - 2r + x = 0$

so,

$\underline{\underline{x = 2r - q}}$ a